

Super Symmetric Partners in T^4 -Space

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Abstract Constructing the operators connecting the state of energy associated with super partner Hamiltonians and super partner potentials for a linear harmonic oscillator has been discussed and it is shown that any super symmetric eigen state of one of the super partner potentials in T^4 -space is paired in energy with a symmetric eigen state of the other partner potential.

Keywords Supersymmetry · Superluminal transformation

1 Introduction

During the past few years, there has been continuing interest [1–4] in higher dimensional kinematical models for proper and unified theory of subluminal (bradyon) and superluminal (tachyon) objects [5, 6]. The problem of representation and localization of extended particles and superluminal objects may be solved only by the use of higher dimensional space. Several attempts of extending special theory of relativity to superluminal realm in the usual four-space [7] led to controversies [8–10] and satisfactory theory for tachyons could not be made acceptable so far. Several experimental investigations [11–15] have been shown towards the evidences for the existence of tachyons: particle moving faster than light ($v > c$) (superluminal particles). Still there are doubts (due to lack of experimental verification) [16] for the existence of tachyons and it still needs the necessity of constructing a self-consistent quantum field theory, which might yield their quantum properties relevant for their production and detection. The continuing interest of tachyons showed that these particles are not in contradiction to the special theory of relativity. A basic disagreement in the various models of tachyons was the spin—statistics relationship. Tanaka [17], Dhar-Sudarshan [18], Aron-Sudarshan [19] assumed the spin for tachyons as that of bradyons. On the other hand,

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Feinberg [20], Hamamoto [21] showed that spin-statistics for tachyons is reversed to that for bradyons. Rajput and coworkers [22, 23] described a Lorentz invariant quantum field theory of tachyons of various spins and showed that tachyons are not localized in space. The Scalar and Spinor field theory of free tachyons [24] shows that tachyons are localized in time. The localization space for the description of tachyons is T^4 -space where the role of space and time (and that of momentum and energy) are interchanged on passing from bradyons to tachyons. The initial values of tachyons lie on the hyper plane $x = 0$ while those for bradyons lie on $t = 0$. The space R^4 - (i.e. the localization space for bradyons) and T^4 - (the time representation space for tachyons) demonstrate the structural symmetry between these two and directly lead to space-time duality between superluminal and subluminal objects. Supersymmetry, i.e. the Fermi-Bose symmetry [25–38] is one of the most fascinating discoveries in the history of Physics. The local extension of supersymmetric theories [39–48] (i.e. super gravity) provides a natural framework for the unification of fundamental interactions of elementary particles. It was believed earlier [20] that spin-0 tachyons are quantized only with anti commuting relations but their localization in space creates problem with Lorentz invariance. Now it is made clear that tachyons are localized in time and their localization space is T^4 -space which behaves as that of bradyons do in R^4 -space and as such it is not possible to accelerate directly a particle from R^4 -space to T^4 -space (or subluminal to superluminal) for which we need superluminal Lorentz transformations (SLT's).

Keeping these facts in mind, in this paper we have undertaken the study of the super partner Hamiltonians in T^4 -space, having the identical energy spectrum, expect for the ground states, and all the potentials exhibiting the shape invariance contain the exact solutions. Constructing the operations connecting the states of same energy for super partner Hamiltonians $H_+(t)$ and $H_-(t)$ and supersymmetric partner potentials $V_+(t)$ and $V_-(t)$ in T^4 -space, all bound state wave functions have been calculated from ground state and shape invariant potentials are obtained for supersymmetric linear harmonic oscillator. It has been shown that n th excited state energy of H_+^T is identical to $(n + 1)$ th excited state energy of H_-^T while any supersymmetric eigen state of one of the super partner potentials in T^4 -space is paired in energy with a symmetric eigen state of the other partner potential. We have also constructed a Semi-Unitary Transformation (SUT) to obtain the supersymmetric partner Hamiltonians for a one dimensional harmonic oscillator and it has been demonstrated that under this transformation in T^4 -space the supersymmetric partner H_+^T loses its ground state but its eigen functions constitute a complete orthonormal set in a subspace of full Hilbert space.

2 Bosonic and Fermionic Partners in T^4 -Space

In order to overcome the various problems associated with superluminal Lorentz transformations (SLTs), six-dimensional formalism [49–53] of space time is adopted with the symmetric structure of space and time having three space and three time components of a six dimensional space-time. In this formalism, a subluminal observer \mathcal{O} in the usual $R^4 \equiv (\vec{r}, t)$ space is surrounded by a neighbourhood in which one measures the scalar time $|t| \equiv (t_x^2 + t_y^2 + t_z^2)^{\frac{1}{2}}$ and spatial vector $\vec{r} = (x, y, z)$ out of six independent coordinates (x, y, z, t_x, t_y, t_z) of the six-dimensional space R^6 . On passing from $R^6 = (\vec{r}, \vec{t})$ to $(R^6)' = (\vec{r}', \vec{t}')^T$ (where T stands the tachyon) via imaginary SLT's, the usual of observer in R^6 will appear as $T^4 \equiv (t_x, t_y, t_z, r)^T$ to the observer \mathcal{O} in R^6 . The resulting space for bradyons and tachyons is thus identified as the R^6 - or $M(3, 3)$ space where both space and time as well as energy and momentum are considered as vector quantities. Superluminal

Lorentz transformations (SLTs) between two frames K and K' moving with velocity $v > 1$ are defined in R^6 - or $M(3, 3)$ space as follows;

$$\begin{aligned}
 x' &\longrightarrow \pm t_x, \\
 y' &\longrightarrow \pm t_y, \\
 z' &\longrightarrow \pm\gamma(z - vt), \\
 t'_x &\longrightarrow \pm x, \\
 t'_y &\longrightarrow \pm y, \\
 t'_z &\longrightarrow \pm\gamma(t - vz),
 \end{aligned}
 \tag{1}$$

where $\gamma = (v^2 - 1)^{-\frac{1}{2}}$. These transformations lead to the mixing of space and time co-ordinates for transcendental tachyonic objects, ($|\vec{v}| \rightarrow \infty$) where (1) takes the following form;

$$\begin{aligned}
 +dt_x &\rightarrow dt'_x = dx+, \\
 +dt_y &\rightarrow dt'_y = dy+, \\
 +dt_z &\rightarrow dt'_z = dz+, \\
 -dz &\rightarrow dz' = dt_z-, \\
 -dy &\rightarrow dy' = dt_y-, \\
 -dx &\rightarrow dx' = dt_x-.
 \end{aligned}
 \tag{2}$$

It shows that we may have only two four dimensional slices of R^6 - or $M(3, 3)$ space $(+, +, +, -)$ and $(-, -, -, +)$. When any reference frame describes bradyonic objects it is necessary to describe $M(1, 3) \implies [t, x, y, z](R^4 - space)$ so that the coordinates t_x and t_y are not observed or couple together giving $t = (t_x^2 + t_y^2 + t_z^2)^{\frac{1}{2}}$. On the other hand when a frame describes bradyonic object in frame K , it will describe a tachyonic object (with velocity $|\vec{v}| \rightarrow \infty$) in K' with $M'(1, 3)$ space i.e. $M'(1, 3) = [t'_z, x', y', z'] = [z, t_x, t_y, t_z](T^4 - space)$.

As such, we define $M'(1, 3)$ space as T^4 -space or $M(3, 1)$ space where x and y are not observed but coupled together giving rise to $r = (x^2 + y^2 + z^2)^{1/2}$. As such, the spaces R^4 and T^4 are two observational slices of R^6 - or $M(3, 3)$ space. Unfortunately the space R^6 - or $M(3, 3)$ is not consistent with special theory of relativity and accordingly the subluminal and superluminal Lorentz transformations lose their meaning in R^6 - or $M(3, 3)$ space in the sense that these transformations do not represent either the bradyonic or tachyonic objects in this space. It has been shown earlier [3–6] that the true localizations space for bradyons is R^4 -space while that for tachyons is T^4 -space. So a bradyonic $R^4 = M(1, 3)$ space now maps to a tachyonic $T^4 = M'(3, 1)$ space or vice versa i.e.

$$R^4 = M(1, 3) \xrightarrow{SLT} M'(3, 1) = T^4.
 \tag{3}$$

Hence R^4 , the suitable space for describing bradyonic phenomena, is mapped under SLT's onto the space T^4 which is the space suitable to describe the tachyonic phenomena. In such

formalism we get the following mapping for the position four-vector $\{x_\mu\} = (\vec{r}, -t)$, four-differential $\{\nabla_\mu\} = (\vec{\nabla}, -\partial_t)$ and four-potential $\{A_\mu\} = (\vec{A}, -\phi_e)$ under SLT's as

$$\begin{aligned} \left\{ \vec{r}, -t = (t_x^2 + t_y^2 + t_z^2)^{\frac{1}{2}} \right\} &\longrightarrow \left\{ \vec{r}, -r = (x^2 + y^2 + z^2)^{\frac{1}{2}} \right\}^T, \\ \left\{ \vec{\nabla}_r, -\partial_t = \partial_{t_x}^2 + \partial_{t_y}^2 + \partial_{t_z}^2 \right\} &\longrightarrow \left\{ \vec{\nabla}_l, -\partial_r = \partial_x^2 + \partial_y^2 + \partial_z^2 \right\}^T, \\ \left\{ \vec{A}, -\phi_e = \phi_{ex}^2 + \phi_{ey}^2 + \phi_{ez}^2 \right\} &\longrightarrow \left\{ \vec{\phi}_e, -A = (A_x^2 + A_y^2 + A_z^2)^{\frac{1}{2}} \right\}^T, \end{aligned} \tag{4}$$

where $\vec{\nabla}_r$ are $\vec{\nabla}_l$ differential operators in terms of three spatial and three temporal coordinates, respectively. Applying these mappings on the equations for generalized electric field, magnetic field and Lorentz force associated with dyons, we get the following expressions respectively for generalized electric field, magnetic field and Lorentz force on dyons under superluminal transformations in T^4 -space i.e.;

$$\begin{aligned} \vec{E}^T &= -\frac{\partial \vec{\phi}_e}{\partial r} - \vec{\nabla}_l A + (\vec{\nabla}_l \times \vec{\phi}_g), \\ \vec{H}^T &= \frac{\partial \vec{\phi}_g}{\partial r'} + \vec{\nabla}_l B + (\vec{\nabla}_l \times \vec{\phi}_e), \\ \vec{F}^T &= \text{Real } q * \vec{\psi} * -(\vec{u} \times \vec{\psi} *) \end{aligned} \tag{5}$$

where $\{B_\mu\} = \{\vec{B}, -\phi_g\}$ is introduced [54] as magnetic four-potential beside the electric four-potential $\{A_\mu\}$; and $\vec{\psi} = \vec{E} - i\vec{H}$ is described as generalized electromagnetic vector field in subluminal frame of reference; and $\vec{u} = \frac{d\vec{r}'}{dr}$ is inverse velocity. These equations show the superluminal fields associated with tachyonic dyons containing both longitudinal and transverse parts. Equation $\vec{F}^T = \text{Real } q * \vec{\psi} * -(\vec{u} \times \vec{\psi} *)$ describes the Lorentz force acting on dyon in T^4 -space. It may be recalled as the force of tachyonic dyon which is similar to that of dyonic force in R^4 -space if we replace the particle velocity $\vec{u} = \frac{d\vec{r}}{dt}$ to that of the inverse velocity $\vec{u} = \frac{d\vec{r}'}{dr}$ alone with the role of space variables is interchanged with time variables. So, the theories of tachyons be better understood in T^4 -space where they behave as bradyons do in R^4 -space. As such in view of their localizability and other quantum properties, we can formulate the supersymmetric theories of tachyons in T^4 -space in the parallel ground of bradyonic supersymmetric theories in R^4 -space. Similarly, the total Hamiltonian H^T of a tachyonic supersymmetric system may be decomposed in to its bosonic part H_B^T and the fermionic part H_F^T such that

$$H^T = H_B^T + H_F^T \tag{6}$$

with

$$H_B^T = -\frac{E^2}{2k} + \frac{1}{2} \{W'(t)\}^2 \tag{7}$$

and

$$H_F^T = \frac{1}{2} W''(t) [\overline{\psi}_T, \psi_T] = -i W''(t) Y_T, \tag{8}$$

where k is the rest mass of tachyon, $W(t)$ is super potential while ψ_T and $\overline{\psi_T}$ are the fermionic variables in T^4 -space describing spin degree of freedom and the operator Y_T is defined as

$$Y_T = \frac{i}{2} \{\overline{\psi_T}, \psi_T\}. \quad (9)$$

In (8), prime and double prime denote first and second order derivatives. Supersymmetric Hamiltonian may also be constructed in terms of non-Hermitian supercharge operators Q and Q^+ as

$$H^T = \frac{1}{2} \{Q, Q^+\} = \frac{1}{2} [QQ^+ + Q^+Q] \quad (10)$$

such that

$$[Q, Q^+] = 0, \quad Q^2 = Q^{+2} = 0 \quad (11)$$

and

$$\begin{aligned} [H^T, Q] &= [H^T, Q^+] = 0; \\ [Y_T, Q] &= -iQ; \\ [Y_T, Q^+] &= iQ^+. \end{aligned} \quad (12)$$

From (6) and (10), the supercharges may readily be constructed accordingly in the following form;

$$Q^+ = [E - iW'(t)]\overline{\psi_T}; \quad Q = [E + iW'(t)]\psi_T. \quad (13)$$

Any state $|B^T\rangle$ satisfies the conditions

$$Q|B^T\rangle = 0; \quad Q^+|B^T\rangle \neq 0 \quad (14)$$

is denoted as the bosonic state for which we have $H^T|B^T\rangle = \frac{1}{2}QQ^+|B^T\rangle$. Similarly, fermionic state $|F^T\rangle$ satisfies the conditions

$$Q^+|F^T\rangle = 0; \quad Q|F^T\rangle \neq 0 \quad (15)$$

which gives $H^T|F^T\rangle = \frac{1}{2}Q^+Q|F^T\rangle$. Using these relations, it may readily be demonstrated that the operator Q transforms states $|F^T\rangle$ in to states $|B^T\rangle$ of the same eigen energy E^T and the operator Q^+ transforms the states $|B^T\rangle$ into states $|F^T\rangle$, i.e.

$$Q|F^T\rangle = E^{T\frac{1}{2}}|B^T\rangle; \quad Q^+|B^T\rangle = E^{T\frac{1}{2}}|F^T\rangle \quad (16)$$

which directly shows $[Q, Q^+] = 0, Q^2 = Q^{+2} = 0$; the supersymmetry between tachyonic fermions and tachyonic bosons with positive momentum eigen values while the negative momentum value corresponds to singularity problem associated with super potential in this supersymmetric model.

Alternatively, supersymmetric quantum mechanics may be worked out in terms of a pair of bosonic Hamiltonians H_-^T and H_+^T which are supersymmetric partners of supersymmetric Hamiltonian i.e.,

$$H^T = H_-^T \oplus H_+^T. \tag{17}$$

In order to construct these super partner Hamiltonians for the system described by (6), let us introduce the potential $V_-(t)$ whose ground state energy has been adjusted to zero with the corresponding ground state wave function given by;

$$\psi_0^{T(-)} = \exp\left[-\int_0^t W(t')dt'\right]. \tag{18}$$

Substituting it in the Schrödinger equation (in the units of $\hbar = 2k = 1$), we get

$$V_-(t) = W^2(t) - W'(t) = \frac{\psi_0^{T''(-)}}{\psi_0^{(-)}}. \tag{19}$$

Hence we have the following Hamiltonian corresponding to the potential,

$$H_-^T = -\frac{d^2}{dt^2} + \frac{\psi_0^{T''(-)}}{\psi_0^{(-)}} = -\frac{d^2}{dt^2} + V_-(t). \tag{20}$$

If the ground state wave function $\psi_0^{T(-)}$ is square integrable then the supersymmetry will be broken. This Hamiltonian may also be written in the following form in terms of bosonic operator \hat{B} and \hat{B}^+ ;

$$H_-^T = \hat{B}^+ \hat{B}, \tag{21}$$

where

$$\begin{aligned} \hat{B} &= \frac{d}{dt} + W(t) = \frac{d}{dt} - \frac{\psi_0^{T''(-)}}{\psi_0^{(-)}}; \\ \hat{B}^+ &= -\frac{d}{dt} + W(t) = -\frac{d}{dt} - \frac{\psi_0^{T''(-)}}{\psi_0^{(-)}}. \end{aligned} \tag{22}$$

Let us introduce the Hamiltonian

$$H_+^T = \hat{B} \hat{B}^+ = -\frac{d^2}{dt^2} + V_+(t) = -\frac{d^2}{dt^2} + W^2(t) - W'(t), \tag{23}$$

where

$$V_+(t) = W^2(t) - W'(t) = V_-(t) + 2W'(t). \tag{24}$$

The potentials V_+ and V_- are called supersymmetric partner potentials and H_+ is the Hamiltonian corresponding to the potential $V_+(t)$. From (13) and (18), we get

$$\begin{aligned} \frac{1}{2} [V_+(t) + V_-(t)] &= W^2(t); \\ [\hat{B}, \hat{B}^+] &= 2W'(t) \end{aligned} \tag{25}$$

showing that $W^2(t)$ is the average of the potential $V_+(t)$ and $V_-(t)$, whereas $W'(t)$ is proportional to the commutator of \hat{B} and \hat{B}^+ . The supersymmetric charges have been defined in terms of operators \hat{B}^+ and \hat{B} . Hence, the Hamiltonians H_+^T and H_-^T are denoted as supersymmetric partners. It can be visualized by introducing supersymmetric charges Q and Q^+ as

$$Q = \sqrt{2} \begin{bmatrix} 0 & 0 \\ B & 0 \end{bmatrix}; \quad Q^+ = \sqrt{2} \begin{bmatrix} 0 & B^+ \\ 0 & 0 \end{bmatrix}. \quad (26)$$

Then we have

$$H_{SUSY}^T = \frac{1}{2} \{Q, Q^+\} = \begin{bmatrix} H_- & 0 \\ 0 & H_+ \end{bmatrix}$$

or

$$[H_{SUSY}^T, Q] = [H_{SUSY}^T, Q^+] = 0$$

or

$$Q^2 = Q^{+2} = 0. \quad (27)$$

For any eigen function $\psi^{T(-)}$ of H_-^T with the corresponding eigen value E^T , we have

$$\hat{B}^+ \hat{B} \psi^{T(-)} = E^T \psi^{T(-)}$$

or

$$H_+^T [\hat{B} \psi^{T(-)}] = E^T [\hat{B} \psi^{T(-)}] \quad (28)$$

which shows that $\hat{B} \psi^{T(-)}$ is the eigen function of H_+^T , with the same eigen value E^T . Similarly it can be demonstrated that if $\psi^{T(+)}$ is an eigen function of H_+^T with the eigen value E^T then $\hat{B}^+ \psi^{T(+)}$ is an eigen function of H_-^T with the same eigen value. Thus H_-^T and H_+^T have the identical energy spectrum except for the ground state of H_-^T . It is also obvious that the operators \hat{B} and \hat{B}^+ connect the states of the same energy for two different supersymmetric partner potentials $V_+(t)$ and $V_-(t)$. Specific relations between spectra of H_- and H_+ are based on the symmetry between the solvable potential and its supersymmetric partner potential.

3 Supersymmetry Partners in T^4 -Space

For supersymmetry one-dimensional Harmonic oscillator we have the super potential

$$W(t) = \Omega t. \quad (29)$$

Then the supersymmetric Hamiltonian is described as

$$H_-^T = \frac{d^2}{dt^2} + \Omega^2 t^2 - \Omega \quad (30)$$

and (19) and (20) give

$$V_-(t) = \Omega^2 t^2 - \Omega. \tag{31}$$

Similarly, (28) leads to

$$V_+(t) = \Omega^2 t^2 + \Omega. \tag{32}$$

From (22), we may then readily get

$$\begin{aligned} B^+ &= -\frac{d}{dt} + \Omega t; \\ B^- &= \frac{d}{dt} + \Omega t \end{aligned} \tag{33}$$

which gives

$$H_+^T = -\frac{d^2}{dt^2} + \Omega^2 t^2 + \Omega. \tag{34}$$

Substituting relation (22) into (18), we get

$$\psi_0^{T(-)} = \left(\frac{\Omega}{\pi}\right)^{\frac{1}{4}} \exp\left[-\frac{\Omega t^2}{2}\right] \tag{35}$$

where the constant $\left(\frac{\Omega}{\pi}\right)^{\frac{1}{4}}$ is the result of the orthonormality of ground state wave function. From (30) and (34), it is obvious that

$$H_-^T \psi_0^{T(-)} = 0$$

i.e.

$$H_0^{T(-)} = 0. \tag{36}$$

Using relations (34) and (35), we get

$$H_+^T \psi_0^{T(-)} = 2\Omega \psi_0^{T(-)}(t) = \Omega' \psi_0^{T(-)}(t), \tag{37}$$

where $\Omega' = 2\Omega$ is the classical frequency of the oscillator. This equation shows that ground state of H_+^T is not the zero energy state i.e. $E_0^{T(+)} = \Omega'$. First excited state $\psi_1^{T(-)}$ of H_-^T may be obtained by the raising operator B^+ i.e.

$$\begin{aligned} \psi_1^{T(-)}(t) &= B^+ \psi_0^{T(-)}(t) \\ &= \left[-\frac{d}{dt} + \Omega t\right] \left(\frac{\Omega}{\pi}\right)^{\frac{1}{4}} \exp\left[-\frac{\Omega t^2}{2}\right] \\ &= 2 \left(\frac{\Omega}{\pi}\right)^{\frac{1}{4}} \Omega t \exp\left[-\frac{\Omega t^2}{2}\right]. \end{aligned} \tag{38}$$

Then we get

$$H_-^T \psi_1^{T(-)} = \Omega' \psi_1^{T(-)}(t) \tag{39}$$

i.e.

$$E_1^{T(-)} = \Omega'.$$

Equations (38) and (39) yield

$$E_0^{T(+)} = E_1^{T(-)} \quad (40)$$

i.e. ground state energy of H_+^T is the first excited state energy of H_-^T . Similarly, we get

$$H_+^T \psi_1^{T(-)}(t) = 2\Omega' \psi_1^{T(-)}(t) \quad (41)$$

i.e.

$$E_1^{T(+)} = 2\Omega'.$$

Second excited state of H_-^T may be constructed as follows;

$$\psi_2^{T(-)}(t) = \frac{B^+}{2} \psi_1^{T(-)}(t) = 2 \left(\frac{\Omega}{\pi} \right)^{\frac{1}{4}} \Omega [-1 + 2\Omega t^2] \exp \left[-\frac{\Omega t^2}{2} \right] \quad (42)$$

for which we have

$$\begin{aligned} H_-^T \psi_2^{T(-)}(t) &= 2\Omega' \psi_2^{T(-)}(t), \\ E_2^{T(-)} &= 2\Omega', \\ H_-^T \psi_2^{T(-)}(t) &= 3\Omega' \psi_2^{T(-)}(t) \end{aligned} \quad (43)$$

showing that first excited state energy of H_+^T is state $\psi_1^{T(-)}$ is identical to the second excited energy of H_-^T in the state $\psi_2^{T(-)}$. Generalizing these results, it may be inferred that with respect to the states $\psi_n^{T(-)}$, the n th excited state energy of H_+^T is identical to the $(n+1)$ th excited state energy of H_-^T . In other words, any state $\psi_n^{T(-)}$ of the super partner Hamiltonian H_-^T corresponding to the eigen value E_n^T of harmonic oscillator is also the eigen state of H_+^T with the corresponding eigen value E_{n+1}^T .

Let us construct the eigen state $\psi_n^{T(+)}(t)$ of H_+^T from the states $\psi_n^{T(-)}(t)$ of H_-^T in the following form;

$$\begin{aligned} \psi_n^{T(+)}(t) &= \frac{1}{\sqrt{E_{n+1}^{T(-)}}} B \psi_{n+1}^{T(-)}(t); \\ \psi_0^{T(+)}(t) &= \frac{1}{\sqrt{E_1^{T(-)}}} \left(\frac{d}{dt} + \Omega t \right) \psi_1^{T(-)}(t) \end{aligned} \quad (44)$$

which may be written in the following form by using (18);

$$\psi_0^{T(+)}(t) = \left(\frac{\Omega}{\pi} \right)^{\frac{1}{4}} \sqrt{2\Omega} \exp \left[-\frac{\Omega t^2}{2} \right]. \quad (45)$$

Then we get,

$$H_+^T \psi_0^{T(+)} = E_0^{T(+)} \Omega' \psi_0^{T(+)}(t) = \Omega' = E_1^{T(-)}. \quad (46)$$

Similarly, we get

$$\begin{aligned} \psi_1^{T(+)}(t) &= \left(\frac{\Omega}{\pi}\right)^{\frac{1}{4}} 3(\Omega)^{\frac{3}{2}} t \exp\left[-\frac{\Omega t^2}{2}\right]; \\ \psi_2^{T(+)}(t) &= 2\left(\frac{\Omega}{\pi}\right)^{\frac{1}{4}} (\Omega)^{\frac{3}{2}} [-1 + 2\Omega t^2] \exp\left[-\frac{\Omega t^2}{2}\right]; \end{aligned} \tag{47}$$

which gives rise to

$$H_+^T \psi_1^{T(+)} = 2\Omega' \psi_1^{T(+)}(t); \quad E_1^{T(+)} = 2\Omega' = E_2^{T(-)} \tag{48}$$

and

$$H_+^T \psi_2^{T(+)} = 3\Omega' \psi_2^{T(+)}(t); \quad E_2^{T(+)} = 3\Omega' = E_3^{T(-)}. \tag{49}$$

Generalizing these results, we may infer that

$$E_n^{T(+)} = E_{n+1}^{T(-)} \tag{50}$$

which shows that n th excited state energy of H_+^T is identical to $(n + 1)$ th excited state energy of H_-^T . In other words, we may infer that if $\psi_n^{T(-)}$ is the n th eigen state of H_-^T with the corresponding eigen value $E_n^{T(-)}$ then $\frac{1}{\sqrt{E_n^{T(-)}}} B \psi_n^{T(-)}(t)$ is the $(n - 1)$ th eigen state of H_+^T with the energy $E_n^{T(+)}$. It is obvious from (34) and (45); (37), (38) and (42); and (47) that for supersymmetry harmonic oscillator we have

$$\psi_n^{T(+)} \propto \psi_n^{T(-)} \tag{51}$$

with the corresponding energy eigen values satisfying condition (50). From (31) and (32) we get the following conditions for the supersymmetry partner potentials for one-dimensional harmonic oscillator

$$V_{\pm}(t) = V_{\pm}(-t). \tag{52}$$

Any anti-symmetric eigen state of one of these potentials is paired in energy with the symmetric eigen state of the other equations (34), (42), (45) and (47) shows that all eigen functions of H_+^T and H_-^T will go to non-zero constant as $t \rightarrow 0$ and hence these are unacceptable as physical states of $V_{\pm}(t)$. These unacceptable solutions of both these super partner potentials are paired in energy and hence the degeneracy theorem holds for $V_{\pm}(t)$ maintaining the supersymmetry of the system.

4 Discussion

Equation (5) for Lorentz force acting on dyon interacting with generalized superluminal electromagnetic fields in T^4 -space is similar to that of a dyon interacting with generalized subluminal electromagnetic fields in R^4 -space except that the role of velocity $\vec{u} = \frac{d\vec{r}'}{dt}$ has been changed with the inverse velocity $\overleftarrow{u} = \frac{d\vec{r}'}{dr}$ and consequently the role of space and time variables are interchanged on passing from subluminal to superluminal realm via

SLT's and thus suggests that tachyonic dyons are localized in time with their fields having rotational symmetry in temporal planes and their motion is bi-dimensional in time.

It is emphasized that there are two observations slices, R^4 - or $M(1, 3)$ (one time and three space) and T^4 - or $M(3, 1)$ (three time and one space), in the existing R^6 -space, $M(3, 3)$ (three time and three space). The transformations for transcendent velocity ($v \rightarrow \infty$) reveal that the tachyons and the bradyons cannot be described individually in the R^6 -space. As such, the equivalence of space and time and the necessity of a four-dimensional Minkowski space to describe any physical event consistent with the special theory of relativity, the T^4 -space have been constructed as the natural space for the description of the tachyons, whereas a bradyon is described in the usual R^4 -space. It has been stressed that a bradyonic R^4 -space maps under SLT's to a tachyonic T^4 -space and on passing from a bradyon to a tachyon via SLT's, the T^4 -space becomes the representation space of the observables while the R^4 -space represents the internal space, where only internal degree for bradyons are represented. It means that the usual pseudo-Euclidean group, E_3 of space-time representation of the Poincare group will be visualized as the usual rotation group in the three time planes in the T^4 -space. Further an object moving with velocity, $v < 1$ (bradyonic) in subspace R^4 (forward velocity $v = \frac{dt}{dr}$) appears to be moving with an inverse velocity ($u = \frac{dr}{dt}$) with $u > 1$. It may also be emphasized that for all-purpose the superluminal fields (tachyonic) in the time-energy representation (T^4 -space) behave as subluminal (bradyonic) fields in space-momentum representation (R^4 -space). Equations (6–8) show the total supersymmetric Hamiltonian in T^4 -space associated with tachyons in terms of energy eigen values and super potentials. Equation (16) shows that the supersymmetry between tachyonic fermions and tachyonic bosons with positive momentum eigen values while the negative momentum values corresponds to singularity problem associated with super potential in the supersymmetric model. Constructing the potential $V_-(t)$ in the form given by (19) with the ground state energy vanishing and the ground state wave function given by (18) the super-partner Hamiltonian H_-^T has been obtained in the form of (20) and it has been found that its ground state energy identical to ground state momentum of a bradyon is vanishing provided that the wave-function defined by (19) is square integrable.

The Hamiltonian H_+^T (super-partner potential of H_-^T) has been constructed in the form of (23) in terms of potential $V_+(t)$ (super-partner potential of $V_-(t)$) given by (24). The supersymmetric charge operators are introduced by equation (26) in T^4 -space. It has been shown that H_+^T and H_-^T have identical energy (momentum) spectrum in $T^4(R^4)$ -space except for the ground state of a Hamiltonian. Equations (41) and (43) show that the first excited state energy of H_+^T in state $\psi_1^{T(-)}$ is identical to the second excited energy of H_-^T in the state $\psi_2^{T(-)}$. Generalization of these results leads to the inference that with respect to the states $\psi_n^{T(-)}$ the n th excited state energy of H_+^T is identical to the $(n + 1)$ th excited state energy of H_-^T . In other words any state $\psi_n^{T(-)}$ of the super partner Hamiltonian H_-^T corresponding to the eigen value E_n^T of harmonic oscillator is also the eigen state of H_+^T with the corresponding eigen value E_{n+1}^T . Equations (34), (42), (45) and (47) provide that all eigen functions of H_+^T and H_-^T will go to non-zero constant as $t \rightarrow 0$ and hence these are unacceptable as physical states of $V_{\pm}(t)$. These unacceptable solutions of both these super partner potentials are paired in energy and hence the degeneracy theorem holds for $V_{\pm}(t)$ maintaining the supersymmetry of the system in T^4 -space.

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